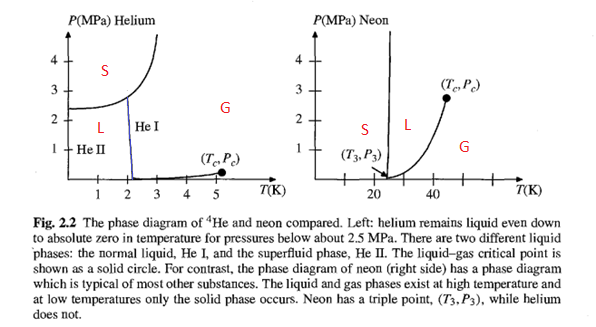
**Thermal Equilibrium Properties**

Here’s a phase diagram,



So gas He smoothliy changes into normal liquid He I (unless it goes through that phase boundary thing at the bottom). There is no distinct phase change at room pressures. And so sometimes people refer to the normal liquid He I as ‘the gas’. There is a phase change between liquid He I and (superfluid) liquid He II, otherwise called, the condensate. And it doesn’t happen all at once as you cross the boundary. Rather, as in the Bose-Einstein transition, once we cross the blue line, the normal liquid begins to turn into condensate until at T = 0, all of the normal liquid is the condensate. He II condensate is GS of interaction. And this state does have some energy, kinetic energy. Though we could call it to zero I guess. Interestingly, except at super high pressures, there is no transition to a solid state; rather, it remains a liquid all the way to T = 0. We can make sense of this. Going back to the Hamiltonian,



and recalling the Leonard Jones potential.



where ε ~ 10kB is the maximum well depth, and σ the distance to the well minimum, and looks like this:

Diagram

Description automatically generated

The 10kB potential well depth suggests that 4He could transition to a liquid/solid state around T ~ 10K, which is so, to O(1). But of course it transitions to a liquid, not a solid. We can make this somewhat plausible. Say the 4He arranged itself in a crystal lattice, say a cubic one for simplicity.

Chart, line chart

Description automatically generated

Then each atom would have 6 nearest neighbors. We could write the Hamiltonian for the crystal as:



where the sum over the potential term is the sum over n.n. So for each i, j would range over 6 values. And we have to multiply by ½ because each bond’s energy is shared with the two atoms connected by it. Can see that we’d overcount the potential energy by factor of two if we did not multipy by ½. Can see this is true in special case where number of nearest neighbors is N-1, i.e., all other atoms in the substance, ‘cause then this just reduces to: (1/2)ΣijVLJ(ri – rj). Anyway. So each atom will feel the potential of each of its 6 neighbors. We can aproximate each potential as a spring potential. So let’s do a harmonic approximation of the potential about its minimum. So we need the second derivative,



So the potential looks like,



where -ε is the well depth of course. So we can approximate the total potential as constituting 2 spring forces in each direction, on either side of our given ith atom. So basically we have:



So this is just a harmonic oscillator situation. The GS energy of our given ith atom would be:



So there is a positive amount of energy per atom, and consequently it wouldn’t be in a bound state. Therefore it’d be a liquid, rather than a solid. Turns out if we repeat the calculation for Ne, we find that the zero point energy is less than the potential well energy, and so a solid state does seem feasible. Neon has 20 nucleons, so…



There we go. One more thing…in the Feynman excitations file we estimate the GS energy as being due to the potential well + the kinetic energy of a particle in a box, rather than a HO. I’m not sure if making this distinction is warranted, but maybe we can say so if we posit that since in the true GS, no collective motion occurs, there shouldn’t be harmonic forces acting between particles, whereas at T = couple Kelvin, we are in the normal liquid, and we do have acoustic oscillations, and so harmonic forces are present??????

The fact that we need quantum mechanics to describe the liquid state is also evinced from the fact that at this temperature, energies are low enough that the thermal wavelength dominates the length scales. For 4He, we find:



So λ > σ, say, and we’re in the regime described by QM. In contrast, the λ for Ne is less than the lattice spacing, and so its transition to a solid can be described by Classical Mechanics.